ECE 580, Optimization by Vector Space Methods — Assignment # 2

Issued: January 30 — Due: February 6, 2008

Reading Assignment:

Luenberger, Section 10.2, and begin Chapter 3.

Problems:

7 Let's return to Problem #2 of Assignment 1. Let $\mathsf{X} = L_p[0,1]$, and define three elements of this Banach space, denoted x^1, x^2, and y, with

$$x^1(t) = 1,\ x^2(t) = t,\ y(t) = t^2, \qquad t \in [0,1].$$

For $a \in \mathbb{R}^2$ we denote $\hat{y}^a = a_1 x^1 + a_2 x^2 \in \mathsf{X}$, which is interpreted as an approximation to y. Compute the best approximation: The vector $a^* \in \mathbb{R}^2$ that achieves the minimum, $\min_a \|y - \hat{y}^a\|$.

(i) Solve for $p = 2$ (the Hilbert space case).

(ii) Solve for $p = \infty$. For this it may be easier to consider the representation in terms of the vector b,

$$y(t) - \hat{y}^a(t) = t^2 - a_1 - a_2 t = (t - b_2)^2 - b_1$$

What values of b_1, b_2 will minimize the L_∞ norm?

(iii) Consider the general approximation problem with $y(t) = t^n$, $x^i(t) = t^{i-1}$ for $i = 1, \ldots, n$ and $t \in [0,1]$. For what values of p could you obtain an expression for a^*? You do not have to compute anything, just explain your reasoning.

8 Luenberger Prob. 10.3

9 Luenberger Prob. 10.4