ECE 580, Optimization by Vector Space Methods — Assignment # 3

Issued: February 6 — Due: February 20, 2008

Reading Assignment:

Luenberger, complete Chapter 3, and begin Chapter 5.

Problems:

10 Before completing this exercise take a look at the Riesz Lemma
See for example `http://en.wikipedia.org/wiki/Riesz's_lemma`

(i) Let Y be a proper closed subset of the normed linear space X (not necessarily a inner-product space). Then, given $\varepsilon > 0$ there exists $x \in \mathsf{X}$ satisfying $\|x\| = 1$ and $d(x, Y) \geq 1 - \varepsilon$. Furthermore, if Y is finite-dimensional one can choose x so that $d(x, Y) = 1$.

What if X *is a Hilbert space*?

(ii) Let X be an infinite dimensional normed linear space. Show that there is a sequence $\{x_n\} \subset \mathsf{X}$ satisfying $\|x\| = 1$ and $\|x_n - x_m\| \geq 1$ for each n, m.

(iii) Conclude that the closed unit ball is not compact if X is infinite-dimensional.

11 Luenberger Prob. 3.7

12 Luenberger Prob. 3.12

13 Luenberger Prob. 3.21

14 Luenberger Prob. 3.22