**ECE 555** **Assignment # 4**

Issued: February 24 Due: March 10, 2011

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**Reading**: Continue reading CTCN: We will cover Sections 9.1–9.5.

**Exercises:**

10. Consider the linear model $X(t+1) = AX(t) + BU(t) + N(t+1)$, with $\boldsymbol{N}$ i.i.d., zero mean, and finite variance. Consider the quadratic cost,

    $$c(x, u) = \tfrac{1}{2}x'Qx + \tfrac{1}{2}u'Ru$$

    where $R > 0$ and $Q \geq 0$. Apply one step of the VIA algorithm to compute $V_1$ *by hand* when $V_0$ is a quadratic. *Easy, right*?

    Explain why computation is it so much harder for the $\ell_1$ control problem, with $c(x, u) = \sqrt{x'Qx} + \sqrt{u'Ru}$.

11. Recall the controlled queueing model,

    $$Q(t+1) = Q(t) - U(t) + A(t+1) \qquad (1)$$

    in which $\boldsymbol{A}$ is i.i.d., and $U(t), Q(t)$ are non-negative valued. We have considered the cost function $c(x, u) = x + \frac{1}{2}u^2$, and found that the relative value function can be approximated by the fluid value function:

    $$\min_u \{c(x, u) + D_u h\,(x)\} \approx \eta$$

    where $h(x) = kx^p$, with $p = 3/2$, and $k$, $\eta$ are constants.

    See if you can find an approximation for the discounted-cost optimal control problem. If desired, you can take the form given in CTCN: Find a function $h$ such that,

    $$\min_u \{c(x, u) + D_u h\,(x)\} \approx \gamma h(x)$$

    where $\gamma > 0$ (corresponding to discount factor $\beta = (1+\gamma)^{-1}$). This simplifies comparison with the fluid model total-cost problem.

12. Returning once more to (1), in the average cost setting, note that the cost function $c(x, u) = x + \frac{1}{2}u^2$ is not well motivated — *why sum the two costs*?

    Let's consider instead the constrained optimization problem,

    $$\textbf{min} \quad \mathsf{E}[U(\infty)^2] \qquad \textbf{s.t.} \quad \mathsf{E}[X(\infty)] \leq \bar{\eta}$$

    where $\bar{\eta}$ is a pre-specified constraint.

    Approximate the solution to this problem, with $\bar{\eta}$ half the steady-state cost obtained when $U(t) = 1\{X(t) \geq 1\}$ (see Theorem 3.0.1 in CTCN).

    To solve this problem you must truncate the state space, and you should assume that $(U(k), Q(k), A(k))$ are restricted to an integer lattice. I'll give you some flexibility in modeling: the marginal of $\boldsymbol{A}$ has mean near 10, and variance between 5 and 25 — for example, you can choose a uniform, or geometric distribution.

    You can solve an LP, or you can compute the solution to the average cost optimization problem with $c(x, u) = \lambda x + \frac{1}{2}u^2$, for various $\lambda > 0$.