

Adaptive Dead-Time Compensation with Application to a Robotic Welding System*

Lyndon J. Brown[†] Sean P. Meyn[†] Robert A. Weber[‡]

July 15, 1997

Keywords: dead time compensation, adaptive control, process control, robotic welding, stochastic control

Abstract

The paper proposes a paradigm for control design suitable for poorly modeled plants with significant dead-time.

An adaptive dead-time compensator is developed which is substantially different to the standard Smith predictor, and more generally applicable in practice. A framework is developed for selecting the controller parameters using now standard design techniques such as μ -synthesis. It is argued that adaptation allows for exact set point matching without the need for integration in the control law. Stability and convergence results are established for the resulting closed loop system equations. The proposed adaptive dead-time compensator is compared with a standard robust controller, and an adaptive pole placement controller through experiments on a gas metal arc welding testbed.

1 Introduction

This paper is concerned with control design for systems which exhibit significant dead-time, and for which an accurate model is not directly available. We will focus the discussion in this paper by considering a specific application: the control of puddle geometry in the gas metal arc welding (GMAW) testbed illustrated in Figure 1. Our goal is to design a controller that can achieve accurate DC tracking of the desired puddle width. Because of significant disturbances, it is desirable to achieve a reasonably large closed loop bandwidth, despite the presence of significant dead-time. A detailed description of the GMAW and some other control approaches may be found in [1, 2].

*This research was partially supported by NSF Grant No. 92-16489, and a grant from the Electric Power Research Institute.

[†]Coordinated Science Laboratory 1308 W. Main Street Urbana, IL 61801

[‡]US Army Construction Engineering Research Laboratories, P.O. Box 9005, Champaign, IL 61826

This system exhibits the following fundamental properties which are found in many other process control applications.

- Significant dead time;
- Significant measurement noise;
- Process parameters that are difficult to measure;
- Process parameters that can show substantial variation over time;
- Non-minimum phase zeros in the process model.

The puddle geometry is measured using a CCD camera which captures the molten puddle extending behind the torch. The weld arc falls in the shadow of the gas cup. The algorithm developed in [3] fits an ellipse to this puddle measurement. The dead time is caused in part by this measurement technique, and delay is also inherent to the thermal processes involved in welding. The dead time decreases slightly as welding speed is increased which results in further complexity and uncertainty.



Figure 1. Gas Metal Arc Welder

In arc welding it is important to accurately control the puddle geometry since this plays a role in determining the quality of a weld [4]. The relationship between puddle geometry and travel rate is subtly affected by many parameters such as plate

temperature, humidity, shield gas composition, and plate and conductor impurities [5]. These quantities may not be easily measured or controlled.

Because of the large amount of parametric uncertainty and time variations inherent in this system, regulation is naturally addressed through the use of adaptation. However, since accurate models of the plant are non-minimum phase, standard adaptive control techniques are not applicable, or may be difficult to implement. In this paper we develop a general adaptive control algorithm for non-minimum phase systems which is shown to successfully solve this specific control problem.

Typical open loop step responses for the welder puddle width are shown in Figure 2. The input for this process is the travel rate of the torch relative to the metal plate which is to be welded. The arc current and the wire feed rate have been held constant. By examining the data around the 17 second mark, one may see that the plant delay is approximately 1.75 s. The sampling time is 0.24 seconds, so the resulting delay is 7 samples. This is confirmed by considering the a priori prediction error when using least squares to fit a fifth order model. A delay of 7 minimizes this criteria for the data collected from the welder.

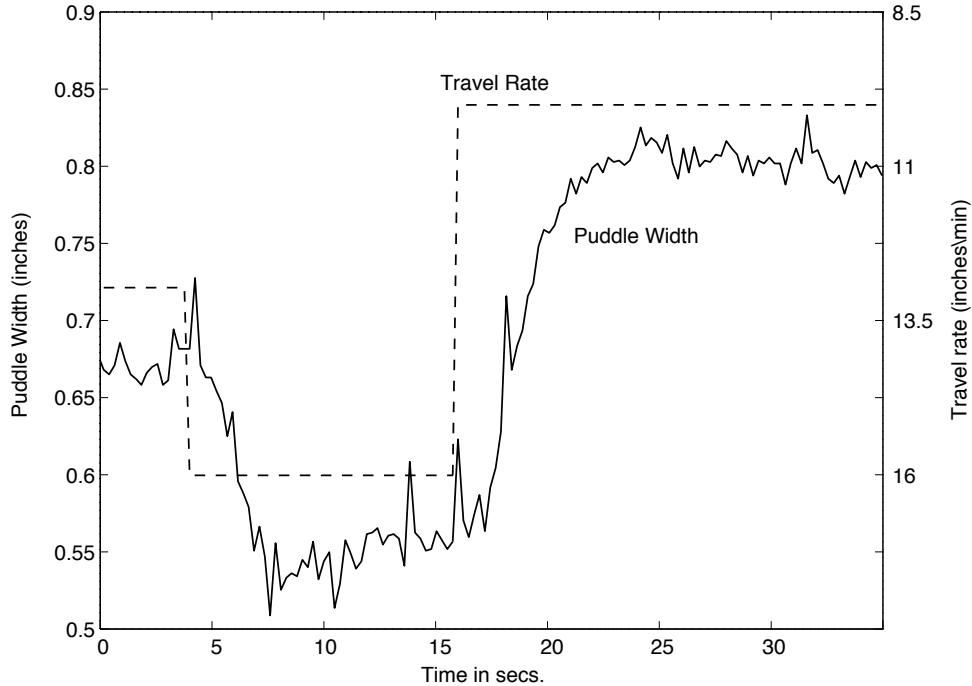


Figure 2. Open Loop Response Of The Welder Puddle Width vs. Travel Rate

A steady state response that results from using a constant travel rate of 13 inches per minute is shown in Figure 3. This figure indicates the level of disturbances and measurement noise encountered with this plant. Here we see that a travel rate of 13 inch/minute corresponds to an average puddle width of 0.525 inches. However, examining the open loop step response Figure 2, one would expect a 13 inch/minute travel rate to produce a 0.675 inch wide puddle. Any controller for this system must be designed to tolerate these types of variation in system behaviour.

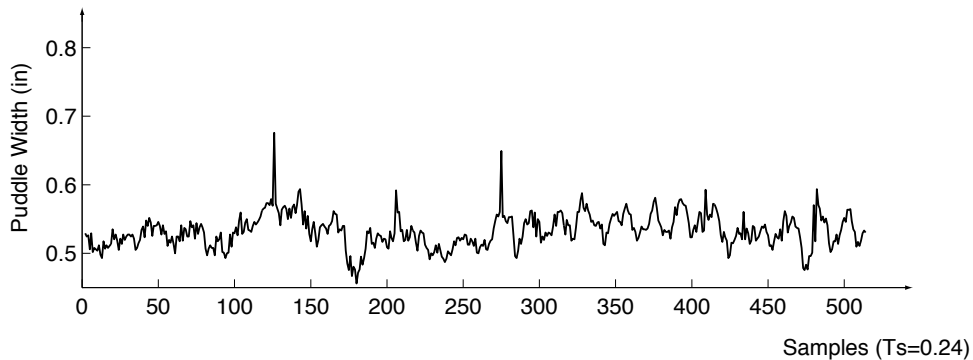


Figure 3. Typical Results with Constant Control

It is well known that phase lag resulting from dead-time makes control design difficult if a large closed loop bandwidth is desired. In the 1950's, Smith presented in [6] a method for introducing predictions in the feedback path to eliminate this inherent phase lag. The subsequent literature on the resulting "Smith predictor" and related "dead-time compensators" is vast (see e.g. [7, 8, 9]). The dead-time compensator (DTC) of [6] and others requires an accurate plant model for effective control design, which is unrealistic in a system such as the GMAW testbed. Here we develop a more widely applicable DTC based upon recursive identification. Identification algorithms such as least squares are defined through the minimization of some prediction error and, under mild conditions, it may be shown that this error is small after suitable normalization (see for example [10, 11, 12, 13]). Given these results, it is natural to apply adaptive predictors so that delay can be disregarded in control design.

In summary, the objectives of this paper are to develop a class of control algorithms which address the usual closed loop design goals: (i) adequate disturbance rejection, and (ii) accurate tracking of a reference input. The control algorithms are designed to perform well for poorly modeled, nearly linear systems which exhibit sig-

nificant dead time. The goal (ii) is not of direct relevance in the welding regulation problem, though it is closely related to (i). However, the approaches described here may be applied directly to the control of other physical systems which suffer from input-output delay, where reference tracking may be a more relevant goal.

The paper is organized as follows. In Section 2 we describe a control law based on adaptive DTC, and give conditions that ensure stability and convergence. In Section 3 we discuss implementation issues, and we find that adaptation naturally leads to accurate set point tracking even with no integral action in the control law. Once a DTC has been designed, a control law that ensures good performance for the delay-free plant model must be constructed. In Section 4 we formulate an appropriate robust control problem whose solution allows us to select the desired control law parameters. Section 5 describes the control synthesis procedure followed for the GMAW testbed, and in Section 6 we present real data to illustrate the effectiveness of this approach.

Portions of the results reported here have been previously published in [14, 15, 16].

2 An Adaptive Dead-time Compensator

Here we describe the control law, and a result that describes the behavior of the resulting closed loop system under ideal conditions. Throughout the paper we consider discrete-time system models. Our goal is to make the sampled output $\{y_k\}$ track the bounded reference signal $\{z_k\}$. To achieve this, the control law under consideration in this paper is defined by finding at time k the input u_k which solves the equation

$$Mu_k + N\hat{y}_{k+d} + \delta_k u_k = N_z z_{k+d} + T\hat{v}_k \quad (2.1)$$

where $\{\delta_k\}$ is a small “dither signal” that ensures that the control law is well defined. The output prediction \hat{y}_{k+d} and the disturbance estimate \hat{v}_k will be defined through some recursive identification scheme such as extended least squares.

The polynomial operators M, N and T must be designed so that the delay-free system is well controlled. The polynomial N_z can be fixed, or it may be defined on-line through adaptation. These issues will be discussed in detail below.

Suppose that the plant is described by the delay- d ARMAX model

$$Ay_k = q^d Bu_k + Cv_k, \quad k \in \mathbb{Z}_+, \quad (2.2)$$

where (y_k, u_k, v_k) denote the output, input, and disturbance, respectively, and q denotes the delay operator. Provided that the disturbance $\{v_k\}$ satisfies appropriate

statistical conditions, for a system model of this form it is easy to construct a minimum mean square error d -step ahead predictor. First let L and L_C denote two polynomials of degree at most $d - 1$ which, together with polynomials G and G_C , solve the Diophantine equations

$$AL + q^d G = 1 \quad (2.3)$$

$$AL_C + q^d G_C = C \quad (2.4)$$

The polynomials L_C and L are related by $L_C = (LC)^0$, where for a polynomial $W(q) = \sum w_i q^i$ we define $W^0(q) = \sum_0^{d-1} w_i q^i$, and $q^d W^1(q) = W(q) - W^0(q)$. The mean-square optimal predictions $\{\hat{y}_k : k \geq 0\}$ can then be expressed by either of the two equivalent expressions:

$$\hat{y}_{k+d} = BLu_k + (CL)^1 v_k + Gy_k = y_{k+d} - L_C v_{k+d}.$$

The estimator is mean-square optimal given the measurements $\{u_i, v_i, y_i, i \leq k\}$ and knowledge of the polynomials (A, B, C) . Since in this paper we assume that substantially less information is available, we now define a suboptimal adaptive predictor.

The model (2.2) may be described in the regressor form $y_k = \theta^T \varphi_{k-1}^0 + v_k$, $k \in \mathbb{Z}_+$, where θ is a vector containing the coefficients of the polynomials

$$\theta := (-a^1, \dots, -a^{\ell_a}, b^0, \dots, b^{\ell_b}, c^1, \dots, c^{\ell_c})^T, \quad (2.5)$$

and φ_k^0 is the regressor vector defined as

$$\varphi_k^0 := (y_k, \dots, y_{k-\ell_a+1}, u_{k-d+1}, \dots, u_{k-d-\ell_b+1}, v_k, \dots, v_{k-\ell_c+1})^T. \quad (2.6)$$

To obtain a satisfactory convergence result we initially consider the projected extended least squares algorithm, which recursively defines estimates $\{\hat{\theta}_k\}$ of the parameter θ as follows

$$\hat{\theta}'_k = \hat{\theta}_{k-1} + \frac{P_{k-1} \varphi_{k-d} e_k}{1 + \varphi_{k-1}^T P_{k-1} \varphi_{k-1}} \quad (2.7)$$

$$\hat{\theta}_k = \Pi(\hat{\theta}'_k, P_{k-1}^{-1}) \quad (2.8)$$

$$P_k = P_{k-1} - \frac{P_{k-1} \varphi_{k-1} \varphi_{k-1}^T P_{k-1}}{1 + \varphi_{k-1}^T P_{k-1} \varphi_{k-1}}. \quad (2.9)$$

where $\Pi(\cdot, Z)$ represents the Z -weighted projection onto some compact convex set Θ [17]. The quantities \hat{v}_k and e_k are defined as

$$\hat{v}_k := y_k - \varphi_{k-1}^T \hat{\theta}'_k \quad e_k := y_k - \varphi_{k-1}^T \hat{\theta}_{k-1} \quad (2.10)$$

where the pseudo-regression vector is defined as

$$\varphi_k := (y_k, \dots, y_{k-\ell_a+1}, u_{k-d+1}, \dots, u_{k-d-\ell_b+1}, \hat{v}_k, \dots, \hat{v}_{k-\ell_c+1})^T. \quad (2.11)$$

Corresponding to the vector $\hat{\theta}_k$ are the three polynomial estimates \hat{A}_k , \hat{B}_k , and \hat{C}_k . Define the polynomials \hat{L}_k and \hat{G}_k by solving the Diophantine equation

$$\hat{A}_k \cdot \hat{L}_k + \hat{G}_k q^d = 1, \quad k \in \mathbb{Z}_+, \quad (2.12)$$

where the operator “ \cdot ” denotes polynomial multiplication independent of the time variation of the polynomial coefficients, i.e.

$$\hat{A}_k \cdot \hat{L}_k = \hat{L}_k + \sum_{i=1}^{\ell_a+d-1} \sum_{j=\max(1, i-d+1)}^{\min(\ell_a, i)} a_k^j \hat{L}_k^{i-j} q^i.$$

The output prediction at time k is then defined by

$$\hat{y}_{k+d} = \hat{B}_k \cdot \hat{L}_k u_k + (\hat{C}_k \cdot \hat{L}_k)^1 \hat{v}_k + \hat{G}_k y_k. \quad (2.13)$$

Since the right side of (2.13) contains only terms known at time k , the prediction \hat{y}_{k+d} is known at time k and may be used in the control law (2.1).

We now define the dither term δ_k to complete the definition of the control law. Because the definition of the estimate \hat{y}_{k+d} contains u_k , to solve (2.1) it is necessary to divide by the term $m^0 + n^0 \hat{b}_k^0 + \delta_k$. We thus make the following definition to avoid potentially large feedback gains:

$$\delta_k = \begin{cases} \delta (\log^{\frac{1}{2}} r_k)^{-1} \text{sign}(m^0 + n^0 \hat{b}_k^0); & \text{if } |m^0 + n^0 \hat{b}_k^0| < \delta (\log^{\frac{1}{2}} r_k)^{-1} \\ 0; & \text{otherwise,} \end{cases} \quad k \in \mathbb{Z}_+, \quad (2.14)$$

where $\delta > 0$ is a small constant, and r_k is defined by

$$r_k = \text{trace } P_k^{-1}, \quad k \in \mathbb{Z}_+. \quad (2.15)$$

If we assume for simplicity that the parameter estimates $(\hat{A}_k, \hat{B}_k, \hat{C}_k)$ converge to some constant values $(\hat{A}, \hat{B}, \hat{C})$, then on combining (2.1) with (2.10), the closed loop system may be written

$$(\hat{A}M + \hat{B}N)y_k = N_z \hat{B} z_k + (\hat{C}M + BN(\hat{C}\hat{L})^0 + q^d BT)\hat{v}_k. \quad (2.16)$$

Thus if $\hat{A}M + \hat{B}N$ is stable and \hat{v}_k is small, the system will be well behaved. Alternatively, to conceptualize this control law one may treat the estimated plant as the

true plant. To see why this may be justifiable it is helpful to consider the a posteriori error definition (2.10) which may be rewritten as

$$\widehat{A}_k y_k = q^d \widehat{B}_k u_{k-d} + \widehat{C}_k \widehat{v}_k. \quad (2.17)$$

Since we expect the prediction error \widehat{v}_k to be small, at least in an average sense (c.f. the aforementioned references [10, 11, 12, 13]), we can treat (2.17) as a known time-varying model of the system to be controlled.

To guarantee stability of the closed loop system we will use the following assumptions. Note that (A1) is the requirement that the control law defined by N/M stabilizes the delay-free plant model.

A1 The polynomial $AM + BN$ is Schur;

A2 The initial conditions are deterministic and $r_0 = \text{trace } P_0^{-1} \geq \epsilon$;

A3 The polynomial $C^{-1} - \frac{1}{2}$ is strictly positive real. That is

$$\text{Re} \left\{ \frac{1}{C(e^{i\omega})} - \frac{1}{2} \right\} > 0, \quad \omega \in [0, 2\pi];$$

A4 The disturbance process $\{v_k\}$ is a bounded d -step martingale difference sequence satisfying for $\beta > 2$, $\sigma_v^2 > 0$, and $\gamma_v < \infty$,

$$\begin{aligned} \mathbb{E}[v_{k+d} \mid v_0, \dots, v_k] &= 0 && \text{a.s.} \\ \mathbb{E}[v_{k+d}^2 \mid v_0, \dots, v_k] &= \sigma_v^2 && \text{a.s.} \\ \mathbb{E}[v_{k+d}^\beta \mid v_0, \dots, v_k] &\leq \gamma_v^\beta && \text{a.s.} \end{aligned}$$

A5 The true parameter vector θ lies in the known compact set Θ used in the projection (2.8).

The common assumption that the plant is minimum phase has been replaced by the condition that the delay-free plant model is stabilized with some known control law. If the algorithm is convergent, it is expected that the closed loop poles will be given by the roots of $AM + BN$. Thus in practice it is necessary that these roots correspond to the poles of a well-behaved system. Similar comments apply to most adaptive control schemes. For example, for minimum variance or model reference control, in practice it is necessary that the plant be not just minimum phase, but that its zeros lie a sufficient distance from the unit circle. Further, for model reference

and pole placement controllers it is necessary that the designed closed loop system be realistically compatible with the actual plant.

Using the techniques of [18] we may establish the following result. A proof is included in [19].

Theorem 2.1 *Suppose that conditions (A1)-(A5) are satisfied, and that the control (2.13), (2.1) together with the estimator (2.8), (2.9) is applied to the plant (2.2). Then we have sample mean square stability:*

$$\limsup_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N (y_k^2 + u_k^2) < \infty \quad \text{a.s.},$$

and the performance is sample mean square convergent:

$$\lim_{N \rightarrow \infty} \frac{1}{N} \sum_{k=1}^N \left(y_k - \frac{BN_z}{AM + BN} z_k - \frac{CM + BN(\hat{L}_k \hat{C}_k)^0 + q^d T}{AM + BN} v_k \right)^2 = 0 \quad \text{a.s.}$$

□

Remarks

- The output y_k is not included as an input to the control law. In the ideal case where \hat{y}_{k+d} is the minimum mean square error estimate, providing y_k as an additional measured output provides no additional flexibility in the control law since $y_k = \hat{y}_k + (LC)^0 v_k$.
- The control law given by (2.13) and (2.1) does not require a stabilizable estimate of the plant to determine the control input - only the “dither term” δ_k is required to ensure that the control law is well defined. This is in contrast to many other approaches where a parameter estimate representing a non-minimal model will lead to a discontinuity in the mapping of parameter estimates to the control law. These discontinuities can result in excessive feedback gains [20, 21, 22].
- It is not assumed in Theorem 2.1 that the leading term b^0 is non-zero. Hence some uncertainty in the delay is permitted in this approach.

3 Disturbance Rejection

To implement the controller (2.1) on a physical plant, it is necessary to consider carefully the issue of disturbances. Here we describe in detail the parameter estimation

algorithm which has been applied on the welding system, and discuss methods for efficiently rejecting DC disturbances.

Theorem 2.1 is based on the assumption that the plant to be controlled is linear and time invariant. This is rarely the case in practice. Physical plants age over time; are often non-linear; and unmodeled exogenous disturbances may be present that may be modeled as parametric variations. These issues are frequently the motivation for using adaptive control. To cope with time variations, it is necessary to prevent the adaptation gain from decaying to zero, as it typically does with the least squares algorithm. Several approaches have been proposed in the literature to prevent the adaptation gain P_k from decaying to zero, e.g., forgetting factor algorithms and covariance resetting. We use the Kalman filter estimation algorithm in developing implementations of the control laws described here. The paper [23] examines the different techniques required to establish stability of adaptive control algorithms using the Kalman filter based estimator.

The measurement process for the welding system is subject to significant high frequency noise. This is evident if one examines the data represented in Figure 3. For this reason, the signals used in the estimation algorithm were sent through a low-pass filter. For the adaptive DTC controllers and adaptive pole placement controller, the filter \mathcal{F} was chosen as $\frac{1-.3q}{1-.7q}$. A superscript f indicates that a signal that has been passed through this filter, e.g. $x^f = \mathcal{F}x$.

The parameter estimates used in the welding experiments are given recursively by

$$\begin{aligned}\hat{\theta}_k &= \hat{\theta}_{k-1} + \frac{P_{k-1}\varphi_{k-1}^f e_k}{R + \varphi_{k-1}^{fT} P_{k-1} \varphi_{k-1}^f} \\ P_k &= P_{k-1} - \frac{P_{k-1}\varphi_{k-1}^f \varphi_{k-1}^{fT} P_{k-1}}{R + \varphi_{k-1}^{fT} P_{k-1} \varphi_{k-1}^f} + Q,\end{aligned}$$

where $Q = 0.0001 * I$, $R = 9$; $\varphi_k^f := (y_k^f, \dots, y_{k-4}^f, u_k^f, \dots, u_{k-d-4}^f, \hat{v}_k^f, \dots, \hat{v}_{k-3}^f)^T$, and the error sequence e_k is

$$e_k := y_k^f - (\mathcal{F} - 1)\hat{v}_k - \hat{\theta}_{k-1}^T \varphi_{k-1}^f.$$

The a posteriori prediction error \hat{v}_k is given by (2.10): we do not use the filtered regression vector in the definition of \hat{v}_k , and in this way we differ from the frequency weighted least squares estimator presented in [17]. In this way we avoid an application of \mathcal{F}^{-1} (typically a high pass filter).

Under mild assumptions, including a weak form of persistence of excitation, one may conclude that the Kalman Filter produces bounded error estimates for bounded inputs even without the use of projection [24]. In our implementations with the Kalman filter we have not used projections and have not encountered difficulties. We rely on the abundance of noise in the system to achieve some level of persistence of excitation.

Exact set point tracking is a common design goal in process control applications. This is usually achieved through the use of integral action in the control law. Unfortunately, discrete integral action introduces significant phase lag at low frequencies. The adaptive approach presented in this paper gives an alternative method for achieving set point tracking.

When the signal to noise ratio is reasonably large at DC, the DC component of the a priori prediction error \hat{v} will be small even if the disturbance contains a significant DC component. For the least squares algorithm this follows directly by examining the normal equations. In view of this and equation (2.16) it is natural to abandon the use of integral control, and instead introduce a time varying scaling of the reference signal to guarantee setpoint tracking. In their treatment of the self-tuning regulator, the authors of [25] use similar ideas to demonstrate that overmodeling of the system to be controlled results in perfect rejection of a periodic disturbance.

Define the polynomial N_z as

$$N_z = \frac{1 + \kappa_k}{\kappa_k} \cdot \frac{N(1)}{N'_z(1)} N'_z \quad (3.1)$$

where N'_z is some design polynomial selected to shape the closed loop reference response and

$$\kappa_k = \begin{cases} \frac{\hat{B}_k(1)}{\hat{A}_k(1)} \cdot \frac{N(1)}{M(1)} & \text{if } \left| \frac{\hat{B}_k(1)}{\hat{A}_k(1)} \right| > \epsilon; \\ \text{sign} \left(\frac{\hat{B}_k(1)}{\hat{A}_k(1)} \right) \cdot \epsilon & \text{otherwise,} \end{cases}$$

where $\epsilon > 0$ is constant. Examining (2.17), it is evident that set point tracking is achieved without using an integrator as long as the error \hat{v} is small.

This reasoning has been verified through experiments on the robotic welder. We have seen in this application that the disturbance estimates $\{\hat{v}_k\}$ generated by the Kalman filter algorithm generally contain no DC component, even though the DC component of the disturbance is known to be significant. This is apparent in the results discussed in Section 5: see Figures 9, 12, and 13 below, where reference tracking has

occurred even though a constant disturbance is known to be present. Another example is provided by the following simulation.

The adaptive controller given by (2.1) was implemented on a model of the welder testbed. The block diagram in Figure 4 indicates the appropriate interconnections of the system. The model is defined by $y_{k+6} = T_{tw}(u_k + v_{k+6})$, where the transfer function T_{tw} is given by

$$T_{tw} = 10^{-2} \frac{0.2267 - 0.7560q - 0.7573q^2 - 0.4699q^3 + 0.4298q^4}{1.0000 - 0.2939q - 0.0390q^2 - 0.1000q^3 - 0.0686q^4 - 0.2240q^5}.$$

The noise source $\{v_k\}$ was a pseudo white Gaussian sequence with variance 4 and mean 5. Since the DC gain of this plant is -0.05, this noise source causes an offset of -0.25 in the output.

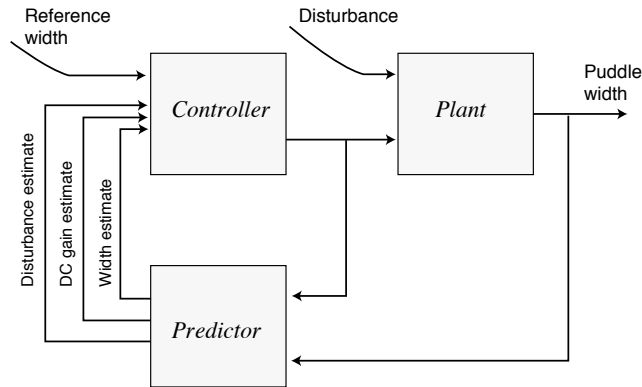


Figure 4. Block Diagram for Simulation with Non-zero-mean Disturbance

The controller was designed using μ -synthesis techniques [26] based on a model of the welder which has a delay of 7 sample periods. The disturbance estimates were not fed back, so the polynomial T is set to zero. The values of M , N and N'_z are given in Table 1 as the Non-Integral DTC μ -controller.

The plant has a delay of only 6, while the estimator uses a model of delay 7. This together with the bias in the disturbance sequence implies that there are unmodeled dynamics present in the parameter estimator.

A typical sample path from the simulations is shown in Figure 5. The width tracks the reference signal given by the dashed line with no constant offsets, despite the fact that there is no integral term explicit in the control law. Figure 6 shows that the disturbance estimates generated by the identification algorithm have little DC content.

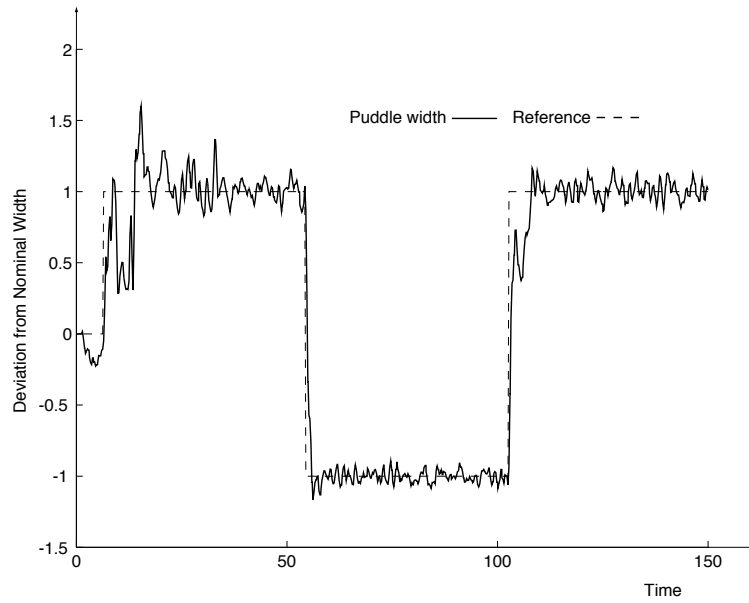


Figure 5. Simulated Controlled Puddle Width in the Presence of Constant Disturbance

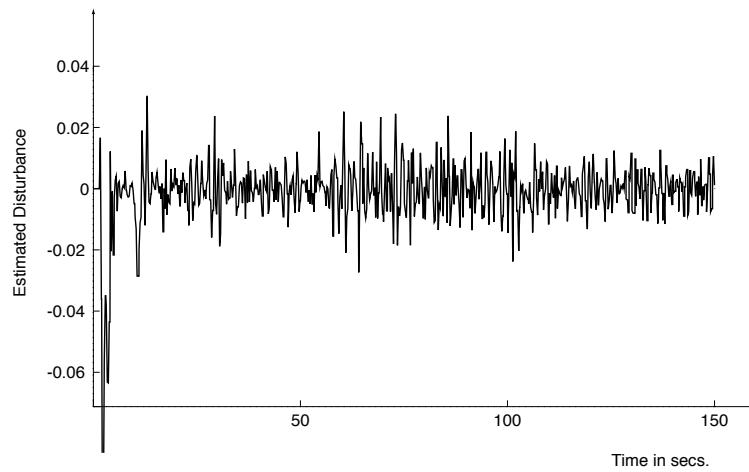


Figure 6. Estimated Disturbance in the Presence of DC Disturbances

4 Nominal Control Selection

Theorem 2.1 shows that we obtain a stable adaptive controller if M/N is chosen so that $AM + BN$ is Schur. Many simple candidate values for M and N may be found if the plant is minimum phase. For example, selecting $M = 0$, $N = A_m$, $N_z = B_m$ and $T = C_m^1 - A_m^1$ results in a model reference adaptive controller (MRAC) similar to the one presented in [27], and identical to that used in [23]. Setting $M = 0$ in this way is a limiting case of high gain feedback. One may thus interpret the control law of [23] and the related minimum variance control law as a combination of prediction with infinite gain feedback (see also [14]).

To design for good closed loop performance we must understand how the incorporation of predictions and disturbance estimates in the control law affects the closed loop system. To develop the control law we consider what can be achieved if the output predictions are ideal and the disturbance estimates are exact. Consider the plant (2.2) together with the (unrealizable) control law

$$M u_k + N(y_{k+d} - L_C v_{k+d}) = N_z z_{k+d} + T v_k. \quad (4.1)$$

Recall that $\hat{y}_{k+d} = y_{k+d} - L_C v_{k+d}$ is the optimal predictor when $\{v_k\}$ is white. The resulting closed loop system becomes

$$(AM + BN)y_{k+d} = BN_z z_{k+d} + (CM + BN L_C + q^d BT)v_{k+d}. \quad (4.2)$$

This is the closed loop system which reaches the asymptotic performance given in Theorem 2.1.

We wish to minimize the error $\{e_k\}$ defined by $e_k = z_k - y_k$, $k \in \mathbb{Z}_+$, while maintaining internal stability of the closed loop system. To obtain a useful synthesis procedure we pose this as an H_∞ robust control problem applied to (4.2). The problem to be solved is then formally stated as follows: Select M , N , N_z , and T so that

$$\{\|W_s e\|_2 < \mu \quad \text{for all } \|v\|_2 < 1, \|z\|_2 < 1\}$$

for all plants in the set

$$\left\{ \frac{B}{A} = \frac{B_o}{A_o} (1 + W_p \Delta) \quad \mid \Delta \in \mathbb{R}(q), \|\Delta\|_\infty < \mu^{-1} \right\}. \quad (4.3)$$

The constant μ and the stable rational weighting functions W_p, W_s are prespecified by the user. Finding the minimal μ and its associated controller is described as the μ *synthesis problem* in the robust control literature.

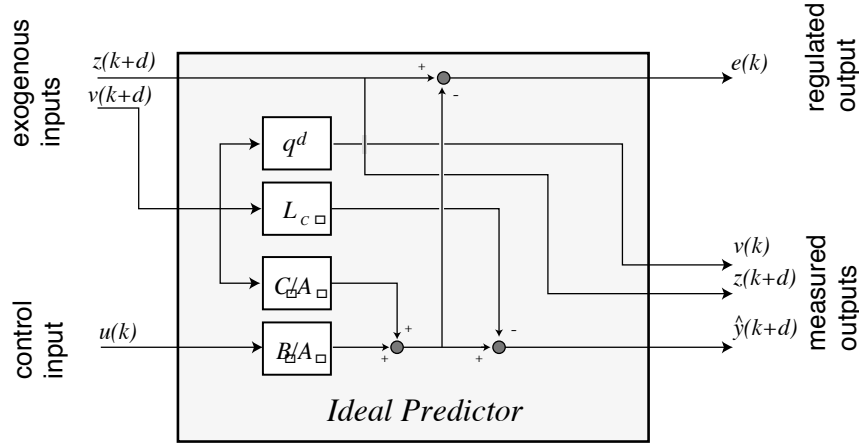


Figure 7. Fictitious Plant for Finding DTC H_∞ Control Parameters

We have chosen a plant model in which we do not model uncertainty in the disturbance transfer function. This uncertainty may be absorbed into the disturbance signal $\{v_k\}$ if the sequence $\{v_k\}$ is only assumed to be bounded and deterministic.

As illustrated in Figure 7, we assume we are able to measure the outputs labeled \hat{y}_{k+d} , z_{k+d} , and v_k , which is indeed possible in the ideal case where the polynomials (A, B, C) are known. Taking the ideal future prediction \hat{y}_{k+d} as a measured quantity may appear artificial. Generally, one measures y_k , and then, based on the nominal model (A_0, B_0) , one generates predictions \hat{y}_{k+d} . Since the true plant is not known this step will introduce errors that will affect the performance and stability of the closed loop system. However, under certain technical conditions the output predictions generated by the estimation algorithm will converge. This is the basis of Theorem 2.1, which shows that for a finite dimensional plant, the adaptive DTC controller results in a stable closed loop system if the compensator $\frac{N}{M}$ stabilizes this fictitious plant.

The solution to this robust performance problem for this fictitious plant gives as desired:

- $AM + BN$ Schur for all plants in (4.3).
- A desirable asymptotic performance level in the ideal case where the conditions of Theorem 2.1 hold, and $(\hat{L}_k \hat{C}_k)^0$ converges to $(LC)^0 = LC$.

An alternative approach for selecting control law parameters to minimize the disturbance response based on H_2 methods is given in [15].

The question may be asked: *if a robust performance problem must be solved, why not solve the robust performance problem for the plant model with the delay included?* Khargonekar and Poolla [28] have shown that introducing non-linearities will not improve the ability to reject disturbance in an H_∞ sense for the unstructured perturbation problem, and this suggests that adaptation can be superfluous. However, this and related results do not apply to the mixed performance/robust stability problem, and generalizations to this setting appear to be unlikely. Moreover, frequently in practice real plants vary with time. Adaptive predictors still perform well even under such non-ideal conditions, and have the potential to re-tune after a change in reference set-point or changing plant dynamics. It is this adaption to changes in the environment which allows this control law to tolerate real-world uncertainties.

5 Control Synthesis for the Welder

Both an adaptive DTC μ -controller and a standard μ -controller were designed for GMAW. To ensure that the closed loop system in each case can tolerate similar levels of plant uncertainty, the plant uncertainty weighting function W_p was kept the same in each design: a first order high-pass filter. The performance weighting function W_s was adjusted in the two designs so that the resulting μ -values were nearly identical. In each case, the function W_s was chosen as a first order low-pass filter. To fix the value of μ it was necessary to reduce the bandwidth of W_s in the non-adaptive case.

Since the software used to design the controllers is intended for continuous time models, the plant model was converted to continuous time using the bilinear transformation. The transformation was done without regard to sampling time, so that frequencies for the weighting function were scaled by a factor of 4. As noted in [29], this transformation preserves the H_∞ norm.

Integral control The weighting function W_p contains a single pole at π rad/sec; a zero at -0.1; and a DC gain of 0.4775. This reflects the uncertainty inherent in developing models at higher frequencies, and the observation that the DC operating characteristics can fluctuate substantially due to lack of control over pre-heating of the plate and other variations that might occur.

The weighting function $W_s = k \frac{s+z}{s+p}$ was chosen to obtain reasonable bandwidth and accurate DC tracking. In the adaptive DTC formulation, the weighting function W_s was chosen to have a pole at approximately 0 (-0.00015). A zero was then placed

DTC μ -Controller			Standard μ -Controller			Non-integral DTC μ -Controller		
M	N	N_z	M	N	N_z	M	N	N_z'
1	0.149067	-0.197975	1	0.167685	-0.108407	1	0.120636	0.112262
-1.481092	-0.114745	0.129176	-1.398545	-0.120413	0.060320	-2.147277	-0.166057	-0.104489
0.685720	0.039435	0.008956	0.907350	0.040070	0.000808	1.442712	0.035976	-0.029126
-0.485176	-0.017841	0.059359	-0.720513	-0.025045	0.029173	-0.430841	0.022576	-0.000811
0.527674	-0.006878	-0.037202	0.631348	-0.006132	-0.017931	0.513471	-0.003802	0.032634
-0.349117	-0.043884	0.008632	-0.405186	-0.043086	0.004018	-0.525547	-0.027655	0.003141
0.164412	0.014823	0.000239	0.231460	0.019223	0	0.181695	0.033101	-0.007373
-0.078742	0.019754	0.001371	-0.215751	0	0	-0.017074	0.010447	0.001785
0.029517	-0.009122	-0.000466	-0.012644	0	0	-0.000251	-0.014521	-0.000120
-0.026105	0.002554	0.000026	-0.126463	0	0	-0.005556	-0.004394	0.000011
0.017895	-0.002523	-0.000344	0.162461	0	0	0.003958	0.000826	0.000096
-0.005715	0.000697	0.000192	-0.062536	0	0	0.000555	0.001289	-0.000004
0.000727	-0.003982	-0.000037	0.009021	0	0	-0.000756	-0.000977	-0.000015
0	0.001378	0				0.000179	0.000152	0.000005
						0.0	0.000400	0.0

Table 1. DTC and Standard μ -Controllers: Coefficients of q^i in the polynomial gains M , N , etc., are written in ascending order

at approximately the desired bandwidth of -0.3rad/s . The gain was chosen to be $k = 0.06$. A controller designed using these weighting functions was found with $\mu = 0.8550$. This control law was of very high order and contained many pole-zero pairs that were equivalent down to 3 significant digits. These near pole-zero pairs were eliminated and the pole at 0.9999 was shifted to 1 to ensure integral action. This controller is given in Table 1.

Since the sensitivity function at DC will be set to zero, this implies that the complementary sensitivity function must be 1 at this frequency, and hence the tolerable multiplicative uncertainty is bounded by 1 at DC. This implies that at a minimum we must know the sign of the DC gain. This is not a restrictive assumption generally, and based on physical considerations it is obvious in the case of the welding system that the DC gain is negative.

In simulations it was seen that the inclusion of the polynomial T did not improve performance, so it was set equal to 0 .

A controller was then developed by solving the standard robust performance problem without adaptation. The same uncertainty weighting function was used, but the zero of the sensitivity weighting function W_s was reduced from -0.3 to -0.075 . This resulted in a value of $\mu = 0.8706$. As before, several nearly identical pole-zero pairs were canceled and the pole near 1 was set equal to 1 . This control is also given in Table 1.

A comparison of the two Bode plots for the resulting control transfer functions N/M is shown in Figure 8. One can see that by eliminating the delay from the plant model we are able to design a controller whose gain at low frequencies is approximately double that of the standard approach. Increasing the controller gain allows a faster response to errors and results in a larger closed loop bandwidth. The adaptive DTC compensator has 50° less phase lead at 2rad/sec, which is to be expected since the dead-time and associated phase lag have been assumed to be removed from the control loop.

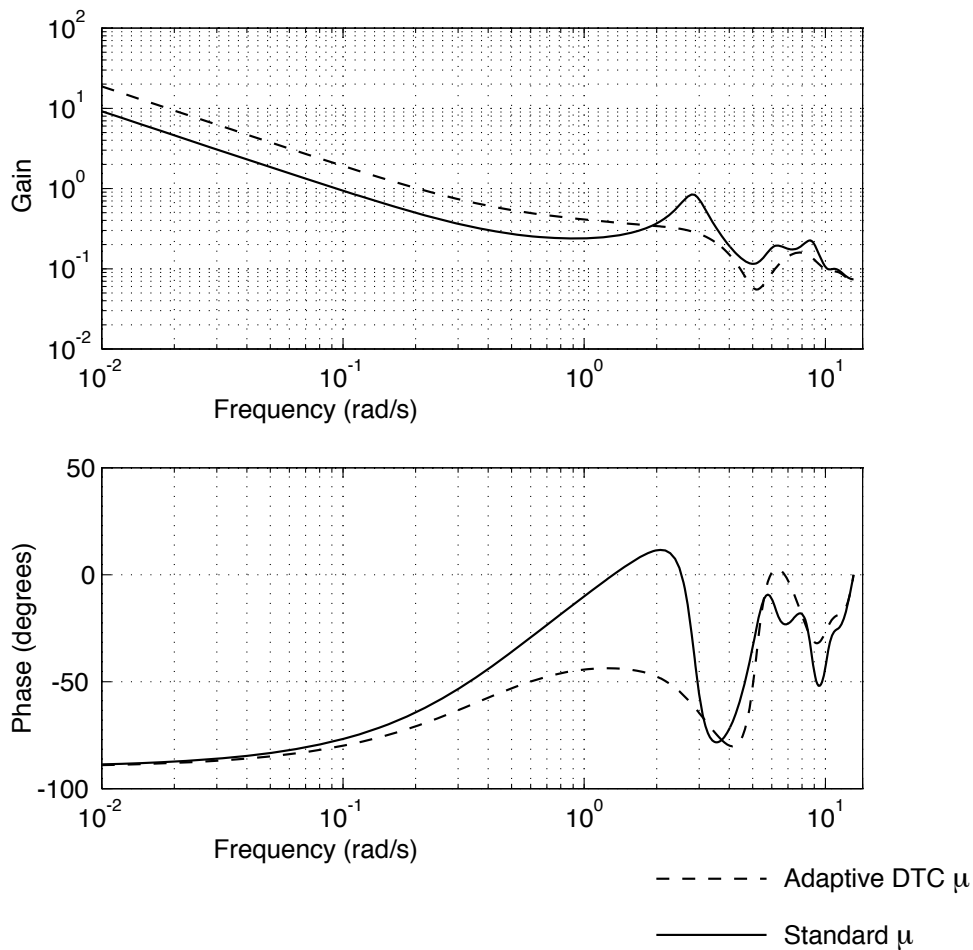


Figure 8. Bode Plot of N/M in two robust designs

Adaptive gain adjustment The sensitivity weighting function for the adaptive DTC controller in the previous design had been selected to ensure set point matching by

placing an integrator in the control law. As discussed in Section 2, set point tracking is naturally achieved through adaptation, so that integration is not necessary. Thus, an alternative adaptive dead time compensator was designed in which the sensitivity weighting was not chosen with a pole at approximately zero and hence significantly less weighting at DC. The same plant perturbation weighting function was used, but the sensitivity weighting function was modified by moving the pole from 0.00 to 0.05. This controller is given in Table 1 where N_z is given by equation (3.1). This resulted in a design with $\mu = 0.698$, a 20% reduction from the previous design.

Pole placement To compare our results with the more traditional adaptive pole placement algorithm, we first need to determine a good choice for the closed loop poles. The number of poles which must be selected is given by $n_A + n_B + d - 1$, where n_A and n_B are the orders of the model polynomials A and B respectively. Since $d = 7$, there is not much to be gained by reducing the order of the polynomials A and B in our nominal model. Thus we continue to use $n_a = 5$ and $n_b = 4$.

It is non-trivial to choose appropriate closed loop poles for this plant. In the experiments reported here, this choice was based on the nominal solution to the standard μ -synthesis problem described above. It might seem probable that this will provide reasonable robustness with good performance. The nominal closed loop solution to the standard μ -problem had 17 poles whereas the pole placement algorithm requires the placement of 15 poles. Two poles were eliminated from the standard μ -controller using Matlab's balanced model reduction routine and the nominal closed loop poles recalculated. To ensure accurate set point tracking, the reference signal was scaled in a manner similar to that discussed for the adaptive dead-time compensator.

The adaptive pole placement algorithm used is described by the equations

$$M_k u_k = N_k (\kappa_k z_{k+d} - y_k)$$

where

$$\begin{aligned} A_m &= M_k \cdot \hat{A}_k + N_k \cdot \hat{B}_k \\ \kappa_k &= \frac{A_m(1)}{N_k(1)\hat{B}_k(1)} \end{aligned}$$

A more traditional approach to achieving set point tracking than the reference scaling shown here would be to solve the Diophantine equation

$$A'_m = M_k \cdot (1 - q) \cdot \hat{A}_k + N_k \cdot \hat{B}_k$$

and use $M_k \cdot (1-q)$ as the denominator of the controller transfer function. This ensures integral control action and hence exact set point tracking if the system is stable. This control law was not implemented on the welding system.

6 Welding Results

Typically there are prices that must be paid for an elegant theory. Due to its generality, it may be too conservative to solve any real-world problems; the assumptions imposed may prevent applicability to all but a small class of problems; and complexity may rule out application. We present here results from the GMAW testbed to show the reader that the results presented here do not fall into any of these traps.

It is necessary to select an operating point about which to linearize in order to approximate the robotic welder plant dynamics with a linear model. Ideally, one wishes to select the operating point such that when the nominal DC input is applied to the plant, the nominal DC output is produced. Unfortunately, the welder's operating points vary substantially in different welds. A nominal 13 inch/minute travel rate can result in nominal puddle widths varying from 0.55 inches to 0.70 inches. Our approach to this problem has been to hold the travel rate constant for the first 30 samples and take the average puddle width during this period as the nominal operating width. This will result in a hopefully small DC error in the noise signal. In order to minimize any transients in the response achieved with the adaptive control law we allow the parameter estimator to operate for an additional 10 samples with the travel rate fixed at 13 inches/minute.

The results of two typical sample runs using the adaptive DTC μ -controller are displayed in Figure 9, and the corresponding travel rates are shown in Figure 10. Comparing Figure 3 and Figure 9, it can be seen that our controller has reduced the effects of disturbances and that steady state set point tracking is highly accurate.

In Figure 10 we see that the travel rates in these two trials were offset by approximately two inches per minute even though the puddle widths were nearly identical. This is in spite of the fact that the two trials were conducted under similar operating conditions with an initial surface temperature of 150° F. However, the pre-heating was accomplished in different manners with different soak times. This likely resulted in different internal temperatures, and is believed to explain the difference in the travel rates. Regardless of its origin, it can be seen that the closed loop system is insensitive to this significant disturbance. In other welds it was found that the closed

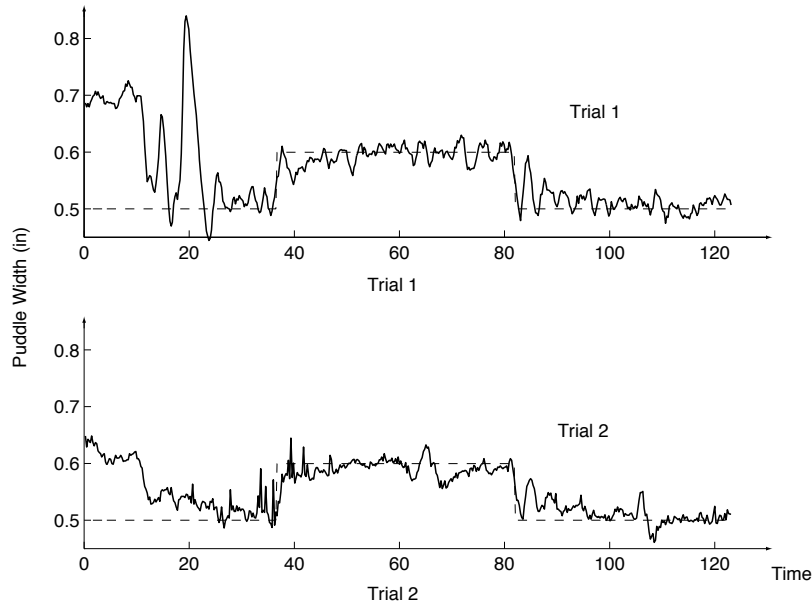


Figure 9. Typical Results with DTC μ -controller

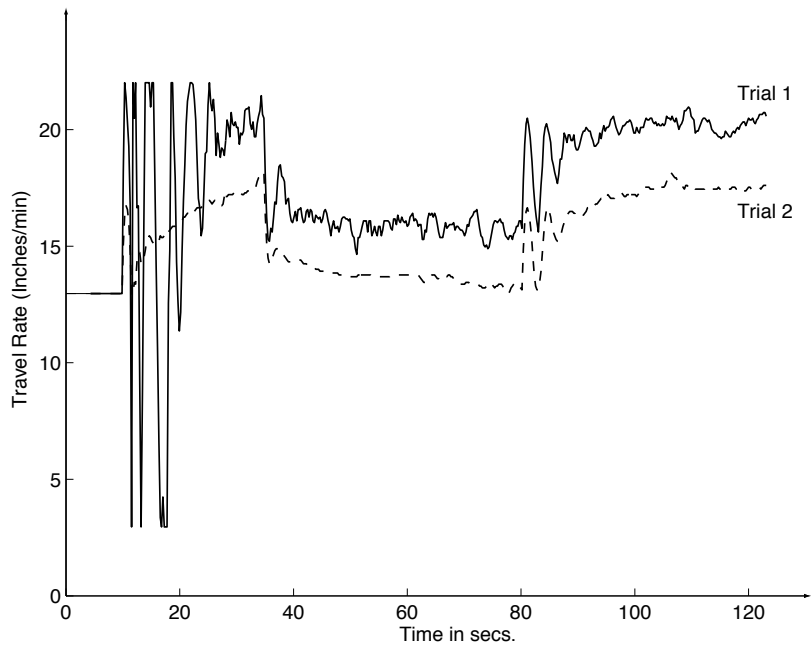


Figure 10. Travel Rates Corresponding These Trials

loop performance was not sensitive to pre-weld external plate temperature. No change in performance was seen when the surface temperature was increased from 150° F to 220° F, and the controlled system behaved acceptably down to room temperature.

The positive and negative step responses for both puddle widths in Figure 9 have substantially different characteristics. The negative step response appears to be more poorly damped than the positive step response. This is again an indication that the controlled GMAW is not a linear plant.

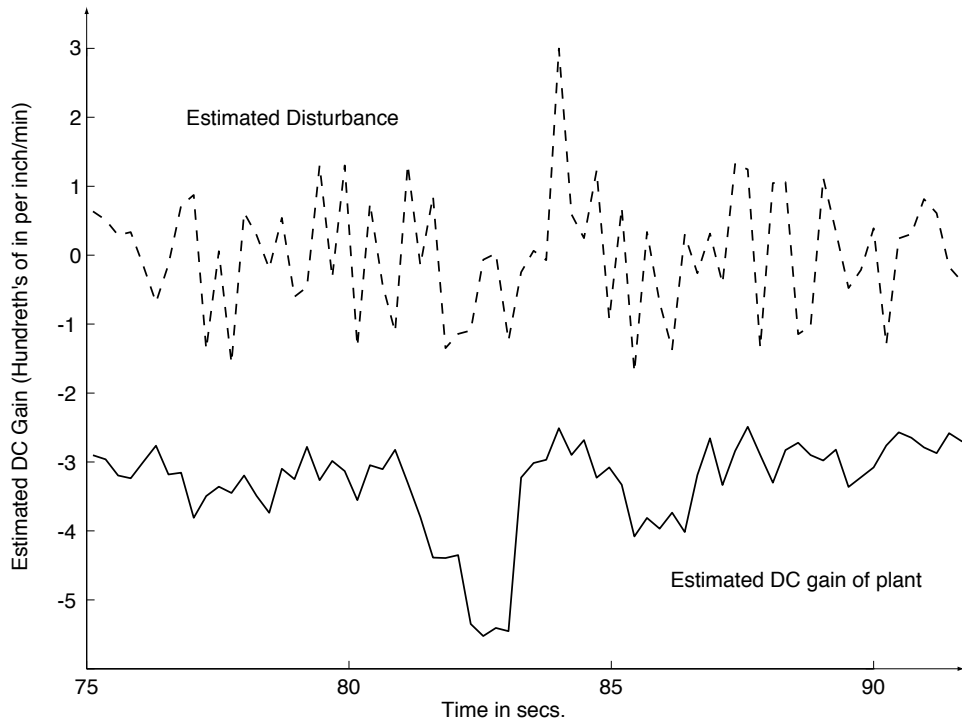


Figure 11. Estimated DC Gain of Plant and Estimated Disturbance

The main difference between Trial 1 and Trial 2 was the choice made for the initial parameter estimate and initial parameter adaptation gain. Trial 1 used an estimate of the parameters based on data collected several months prior to this trial when the welder was displaying substantially different DC characteristics. In accordance with this greater uncertainty in the parameter estimates, the initial adaptation gain was chosen to be very large, $P_0 = 10$. Trial 1 is similar to responses seen when a random choice is used for the initial parameter estimate, and is indicative of the worst transient performance one can expect with this algorithm. Even though the choice of M and N should limit high frequency control action, the travel rate is still

seen to be saturated for the first 10 seconds of operation. This occurred because the initial output predictions were off by at least an order of magnitude. This problem may be reduced by the simple expedient of clipping the predicted values to reasonable levels.

In Trial 2 the initial parameter estimate was the final parameter estimate from the previous weld and the initial adaptation gain was chosen as $P_0 = 0.1$. Recall that the controller does not commence operation until after 10 seconds has passed. Here almost no transient response from the initial parameter estimate error is detectable. This is the response one would expect to see in an industrial setting in which many welds are performed in a single operating period.

It is common practice in industry to reinforce welds by placing a second weld on top of an existing weld. Attempting to achieve consistent puddle width in this situation is a significantly harder problem. Due to the lack of uniformity in the existing weld, much greater fluctuations in the disturbances and process parameters can be expected. Figure 12 demonstrates the results achieved when welding over an existing weld. The severe deviation of the measured puddle width to values below 0.4 at $t = 67s$. was a result of a failure of the measurement system. The measurement system was reset at $t = 68s$, and the subsequent measurements were more accurate. The ability of this control law to recover from substantial disturbances is evident from these plots.

Figure 13 shows the results achieved when the non-integral adaptive DTC μ -controller was employed. Figure 14 represents a close up view of the positive step in Figure 13. This allows a better examination of the step response achieved with the adaptive DTC μ -controller. The controller achieves a reference response rise time of only 4 samples or .96s. We see that, as expected, exact set point tracking has occurred. Since the weighting functions were designed to give this controller a similar bandwidth as the controller with integral action, we see very similar performance in Figure 9 and Figure 13. However the μ value for the non-integral design is 20% lower than the design based on integral control, and hence the non-integral controller should be able to tolerate larger errors in the nominal plant model.

While the robust controller performed well in simulations, it performed unexpectedly poorly when implemented on the welder. As can be seen from the typical response achieved with the robust controller illustrated in Figure 15, the rise time was approximately 8s or 32 samples, and the response was poorly damped. The oscillations suggest that the system may not be robust to parametric variations. This

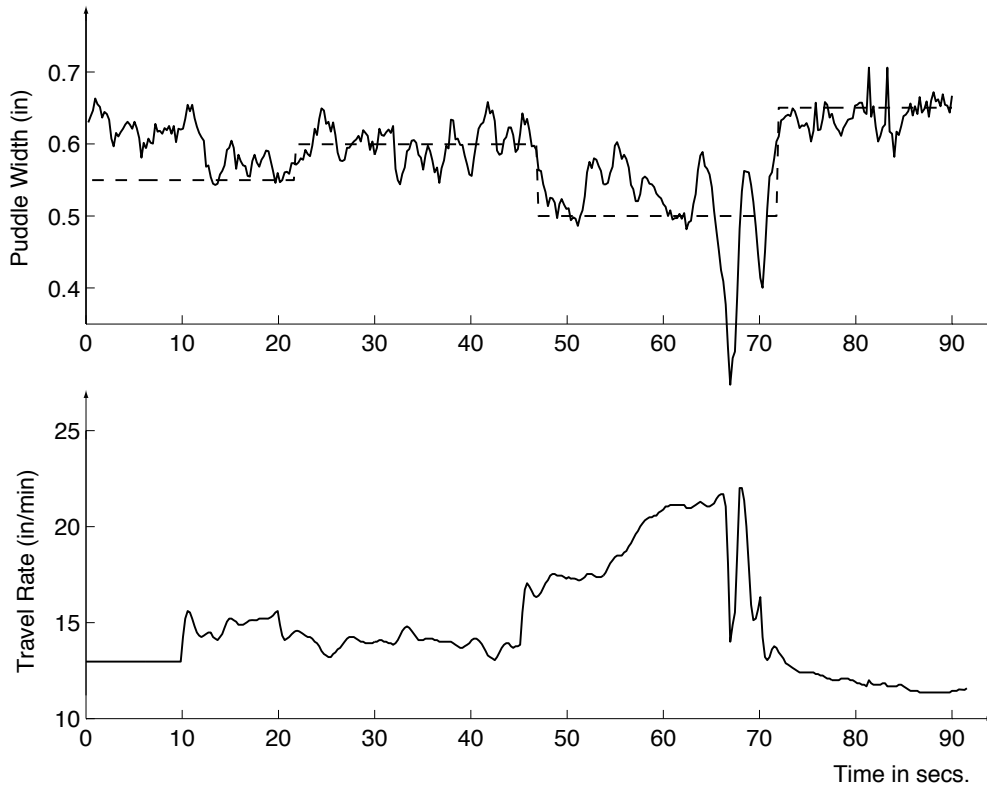


Figure 12. Results when Reinforcing Weld

was verified when a small adjustment in the shield gas composition resulted in an unstable system response.

An examination of Figures 11 and 2 provides an explanation for why the robust controller performed poorly. Considering Figure 11, one would expect the DC gain of the welding system to be about -0.03 inches per inch/minute. This is verified by examining the change in travel rates in Figure 10 with the corresponding changes in puddle width in Figure 9. The data in Figure 2 which was used to construct a plant model for control design was collected under similar operating conditions. From Figure 2 we see that a 6 inch per minute step in the travel rate results in a 0.25 inch change in puddle width, which implies a DC gain of -0.048. In fact the plant model used to design the controllers had a DC gain of -0.052 inches per inch/min. Thus, on the day the data represented in Figure 9 was collected, the welder deviated roughly 42.3% from the plant model at DC. The perturbation weighting of 47.75% combined with the value $\mu = 0.8706$ only allows for deviations of 54.8%. With the introduction

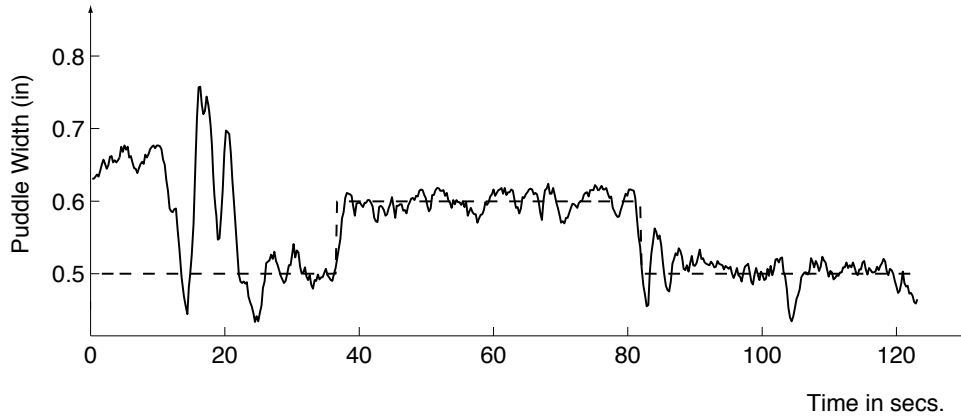


Figure 13. Puddle Width with No Integration in Control Law

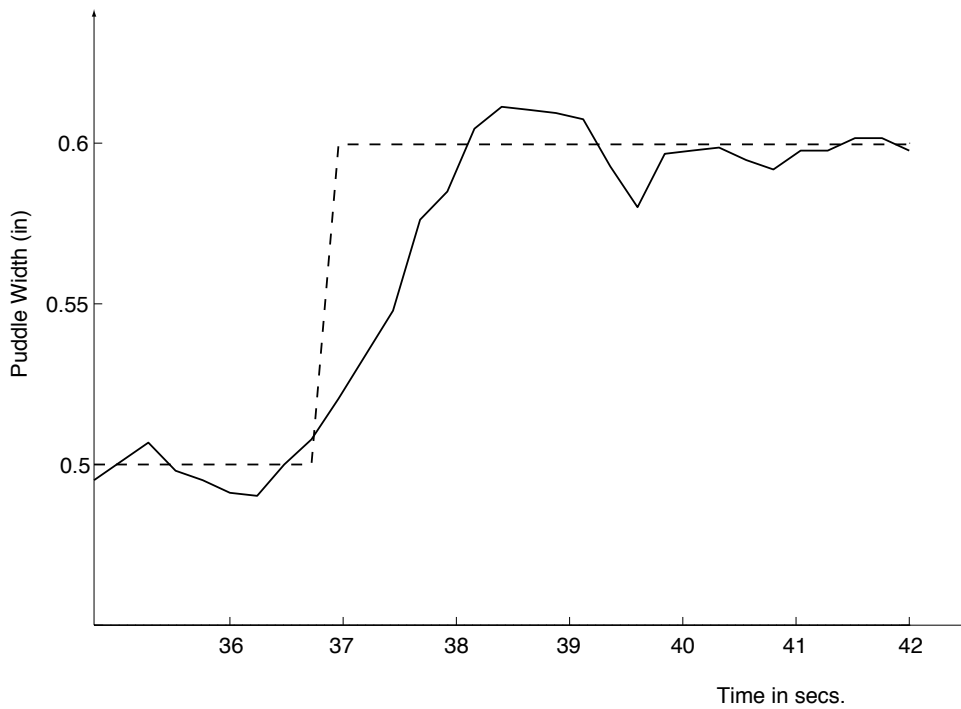


Figure 14. Step Response Achieved With Non-integral DTC μ -controller

of phase shifts at non-zero frequencies, it is reasonable to expect even larger model error at higher frequencies. Thus the design specifications for the robust controller may not have provided for the actual perturbations present in the system.

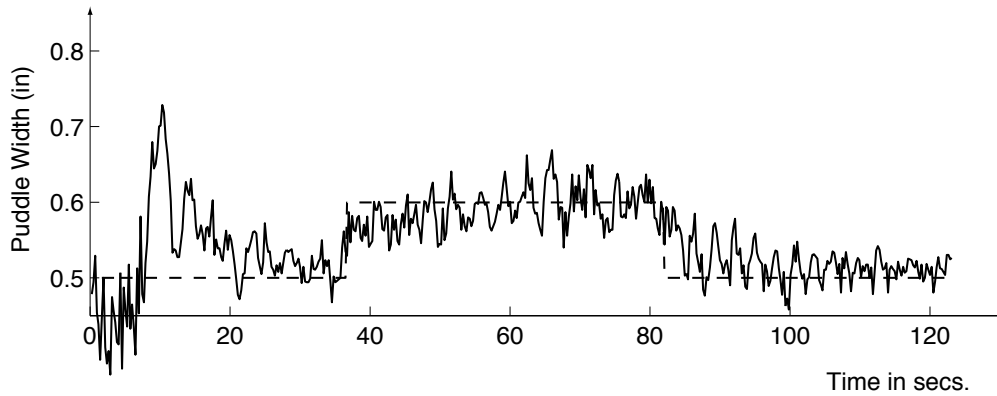


Figure 15. Results with Standard μ -Controller

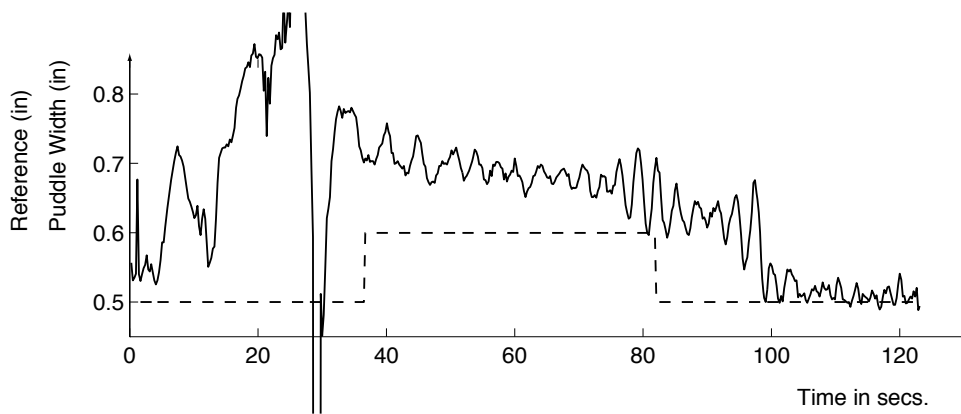


Figure 16. Results with Adaptive Pole Placement Controller

The oscillations seen in Figure 15 could have been predicted from Figure 8 which shows that the standard robust control compensator exhibits a peak in its magnitude plot at approximately 3 radians/second. This is roughly equal to the frequency of the oscillation seen in Figure 15. The compensator used with the adaptive DTC does not possess this undesirable peak.

Typical parameter estimation schemes used in adaptive control such as least squares exhibit faster convergence of output predictions than parameter estimates. Since pole placement algorithms rely on the parameter estimates directly, transient behaviour may persist longer and may be of greater magnitude than for the adaptive dead-time compensator approach. This observation is illustrated in Figure 16, where

a typical puddle width plot resulting from an adaptive pole placement algorithm is shown. This algorithm eventually produced good regulation about the final set point value. However, the entire weld was nearly complete before the closed loop system performed adequately.

Figure 17 shows the travel rate that corresponds to the puddle width in Figure 16. Since the control sequence initially contains significant high frequency content, the estimator is forced to match the plant's frequency response at higher frequencies. Because of the presence of non-linearities and unmodeled dynamics, this may lead to poor matching in lower frequency bands, including at DC.

In [14] the results of applying a PI and an adaptive DTC PI controller were presented. For a puddle width set point of 0.55 inches, reference response rise times of respectively 3.5s and 1.2s were obtained. This is slower than the rise time of 0.96s achieved with the controller presented in this paper. In addition, the PI controller resulted in an unstable system when the operating point was increased to 0.65 inches and the adaptive DTC PI controller also suffered a severe degradation of performance at this operating point. The adaptive DTC μ controller showed no change in performance when the set-point was varied from 0.45in to 0.65in.

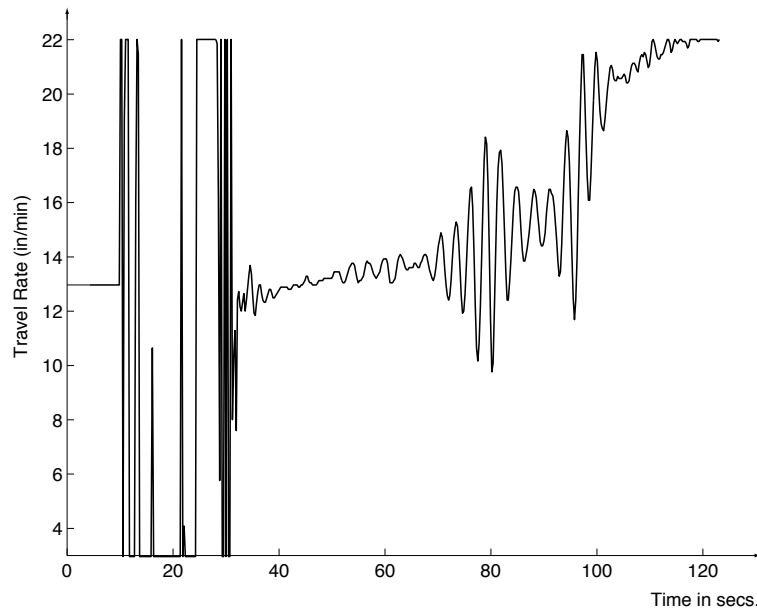


Figure 17. Travel Rate with Adaptive Pole Placement Controller

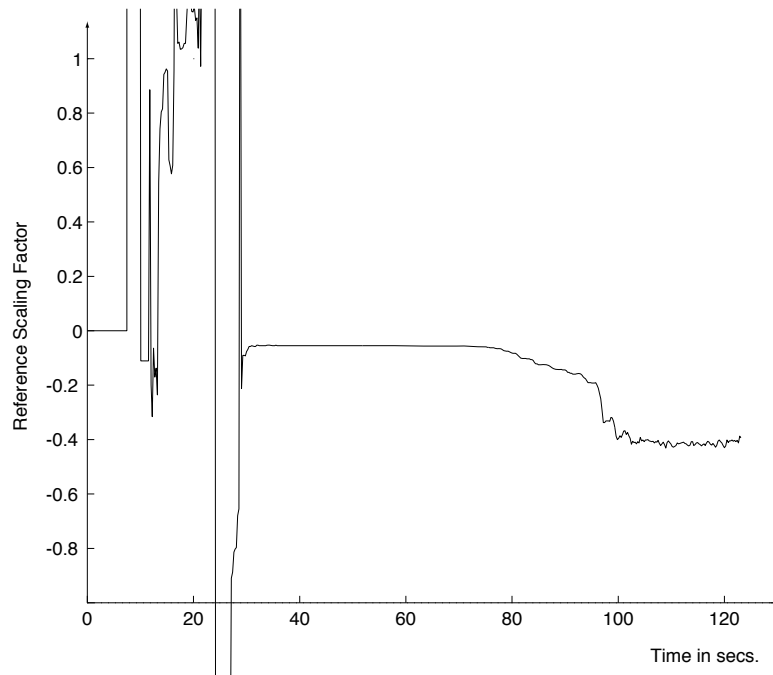


Figure 18. Reference Scaling Constant for Adaptive Pole placement Controller

7 Conclusions & Future Work

One of the most significant differences between the control laws described here and other adaptive control algorithms is that the idea of a *universal controller* has been abandoned. The controller designed for the robotic welding system has been tuned to give excellent performance on 1/2 inch steel plates with a puddle width of 0.6 inches. If with this controller a 1/4 inch or 1 inch plate is substituted, or a substantially different size puddle is required, then closed loop performance may suffer. This is a weakness and also a strength of this approach. Its strength is that prior knowledge such as plate thickness and desired geometry can be easily incorporated into control design allowing maximum performance. Its weakness is of course this lack of “universality”.

If greater tolerance to uncertainty is desired, the performance weighting functions could be redesigned to achieve this goal at the expense of slower responses. Alternatively, the adaptive dead time compensator can be coupled with more complicated control laws than the fixed control law presented here. Such control laws could include adaptive pole-placement, adaptive LQR, or adaptive H_∞ controllers for the delay-free model. To reduce complexity, it is necessary to reduce the bandwidth and also reduce the order of the system model used for identification purposes. To avoid the difficulties we have seen in our experiments, which seem to be inherent to adaptive pole placement and probably also adaptive LQR, it will be necessary to filter the signals more strongly to avoid high frequency excitation. This approach is currently under investigation.

In the future we plan to extend control of the welder to controlling the puddle area and cooling rate as well as the puddle width. We plan to investigate use of the shield gas composition as well as arc current as possible additional control inputs.

There are many other applications which could benefit from adaptive dead-time compensation. Among these are as paper manufacture, temperature control such as in the semiconductor manufacturing industry, and chemical manufacturing.

REFERENCES

- [1] D. E. Henderson, P. V. Kokotović, J. L. Schiano, and D. S. Rhode, “Adaptive control of an arc welding process,” *Control Systems Magazine*, vol. 13, pp. 49–53, February 1993.
- [2] D. Nishar, J. Schiano, W. Perkins, and R. Weber, “Adaptive control of temperature in arc welding,” *IEEE Control Systems Magazine*, vol. 14, pp. 4–12, August 1994.
- [3] R. S. Baheti, “Vision processing and control of robotic arc welding system,” in *Proceedings of the IEEE 24th Conference on Decision and Control*, (Ft. Lauderdale, FL), pp. 1022–1024, 1985.
- [4] American Welding Society, *Handbook of Welding*, vol. 1 of 4. eighth ed., 1987.
- [5] R. D. Stout, *Weldability of Steels*. New York, NY: Welding Research, 1987.
- [6] O. Smith, “Close control of loops with dead time,” *Chemical Engineering Progress*, vol. 53, no. 5, p. 217, 1957.
- [7] K. Åström, “Frequency domain properties of Otto Smith regulators,” *International Journal of Control*, vol. 26, no. 2, pp. 307–314, 1977.
- [8] Z. Palmor, “Stability properties of Smith dead-time compensator controllers,” *International Journal of Control*, vol. 32, no. 6, pp. 937–949, 1980.
- [9] Z.-Q. Wang and S. Skogestad, “Robust control of time-delay systems using the Smith predictor,” *International Journal of Control*, vol. 57, no. 6, pp. 1405–1420, 1993.
- [10] T. L. Lai and C. Z. Wei, “Least squares estimates in stochastic regression with applications to identification and control of dynamic systems,” *Ann. of Math. Statist.*, vol. 10, pp. 154–166, 1982.
- [11] T. L. Lai and C. Z. Wei, “Extended least squares and their applications to adaptive control and prediction in linear systems,” *IEEE Transactions on Automatic Control*, vol. AC-31, pp. 898–906, 1986.
- [12] H. F. Chen and L. Guo, “Convergence rate of least squares identification and adaptive control for stochastic systems,” *International Journal of Control*, vol. 44, pp. 1459–1476, 1986.
- [13] P. R. Kumar and P. P. Varaiya, *Stochastic Systems: Estimation, Identification and Adaptive Control*. Englewood Cliffs, NJ: Prentice-Hall, 1986.

- [14] L. J. Brown and S. P. Meyn, "Prediction and adaptation in PID controller design," in *Proceedings of the 32nd IEEE Proceedings Conference on Decision and Control*, (San Antonio, TX), pp. 1575–1580, December 1993.
- [15] L. J. Brown and S. P. Meyn, "A synthesis of classical and adaptive control," in *Proceedings of the second IEEE Conference on Control Applications*, (Vancouver, British Columbia), September 1993.
- [16] L. J. Brown and S. P. Meyn, "Adaptive dead time compensation with application to robotic welding," in *Proceedings of the 34th IEEE Conference on Decision and Control*, (New Orleans, LA), 1995.
- [17] G. C. Goodwin and K. S. Sin, *Adaptive Filtering, Prediction and Control*. Englewood Cliffs, NJ: Prentice-Hall, 1984.
- [18] W. Ren and P. R. Kumar, "Stochastic adaptive system theory: Recent advances and a reappraisal," in *Foundations of Adaptive Control* (P. V. Kokotović, ed.), pp. 269–307, Springer-Verlag, 1991.
- [19] L. J. Brown, "Adaptive dead time compensation," Technical Report CD-166, University of Illinois, June 1994, and Ph.D Thesis, May, 1996.
- [20] F. Giri, M. M'Saad, L. Dugard, and J. M. Dion, "Robust adaptive regulation with minimal prior knowledge," *IEEE Transactions on Automatic Control*, vol. AC-37, pp. 305–315, March 1992.
- [21] P. D. Larminat and H. Raynaud, "A robust solution to the admissibility problem in indirect adaptive control without persistency of excitation," *International Journal of Adaptive Control and Signal Processing*, vol. 2, pp. 95–110, 1988.
- [22] R. H. Middleton, G. C. Goodwin, D. J. Hill, and D. Q. Mayne, "Design issues in adaptive control," *IEEE Transactions on Automatic Control*, vol. AC-33, no. 1, pp. 50–58, 1988.
- [23] S. P. Meyn and L. J. Brown, "Model reference adaptive control of time varying and stochastic systems," *IEEE Trans. Automat. Control*, vol. 38, no. 12, 1993.
- [24] L. Guo, "Estimating time-varying parameters by the Kalman filter based algorithm: Stability and convergence," *IEEE Transactions on Automatic Control*, vol. AC-35, pp. 141–147, February 1990.
- [25] B. F. La Scala and R. R. Bitmead, "A self-tuning regulator for vibration control," *Proceedings of the 34th Conference on Decision and Control*, vol. 2 of 4, pp. 1141–1146, December 1995.

- [26] G. J. Balas, J. C. Doyle, K. Glover, A. Packard, and R. Smith, *μ -Analysis and Synthesis Toolbox*. Natick, MA: The MathWorks, Inc., 1991.
- [27] G. C. Graham and D. Q. Mayne, "Continuous-time stochastic model reference adaptive control," *IEEE Transactions on Automatic Control*, vol. 36, pp. 1254–1263, November 1991.
- [28] P. Khargonekar and K. Poolla, "Uniformly optimal control of linear time-invariant plants: Nonlinear time-varying controllers," *Systems & Control Letters*, vol. 6, pp. 303–308, January 1986.
- [29] T. Chen and B. Francis, *Sampled-Data Control Systems*. Springer-Verlag, March 1994.